Scientific Writing/Presentation
Non-inertial frames

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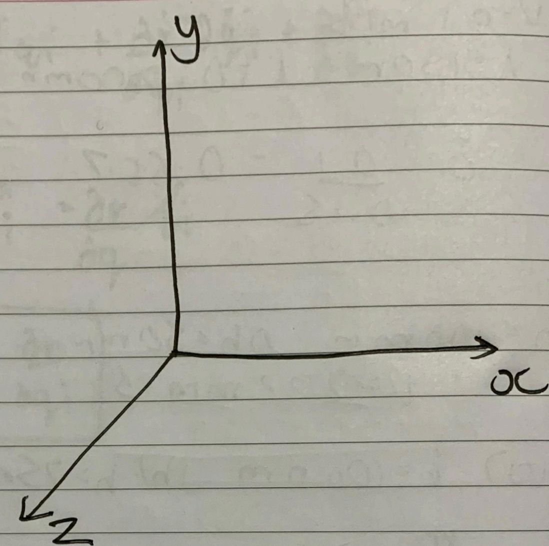
Introduction

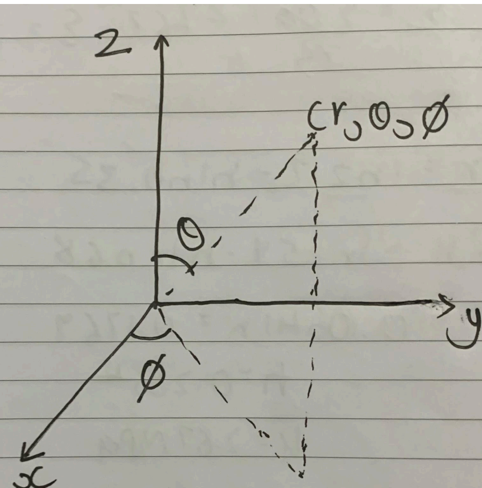
To start understanding non-inertial frames, we will first need to understand frames and how we take the reference of frames in both classical and relativistic physics.

Frame of Reference

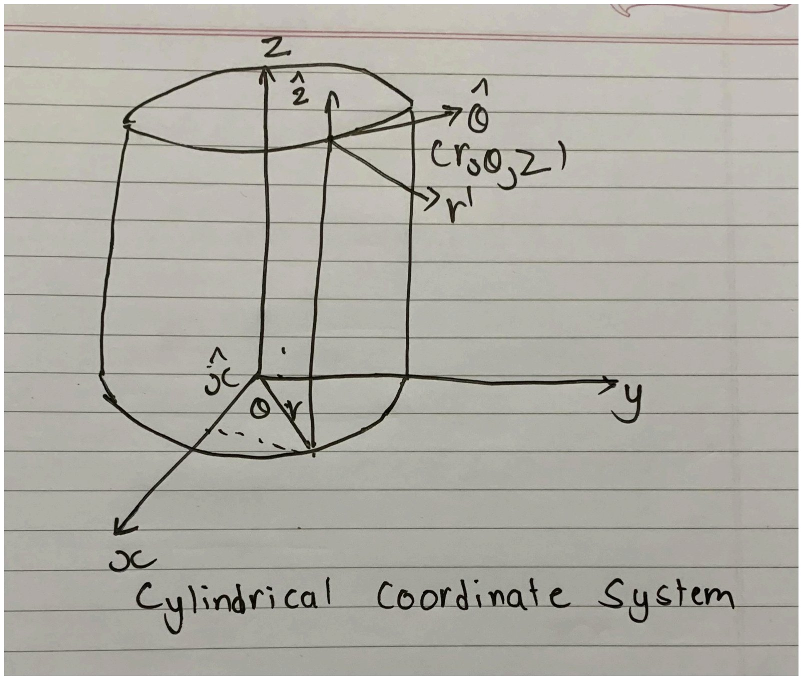
Frames of Reference are assumed for a given system of particles. We choose frames in such a way that motions and forces on the system of particles that we are describing can be determined in the simplest form possible. Frames of Reference consist of a system of coordinates and a set of reference points which we usually call the origin of the reference frame. Different reference frames can have different systems of coordinates. Some of the most basic examples of systems of coordinates are cartesian coordinates, spherical polar coordinates, cylindrical coordinates, etc. The images of the coordinates are attached below.



Cartesian Coordinate System

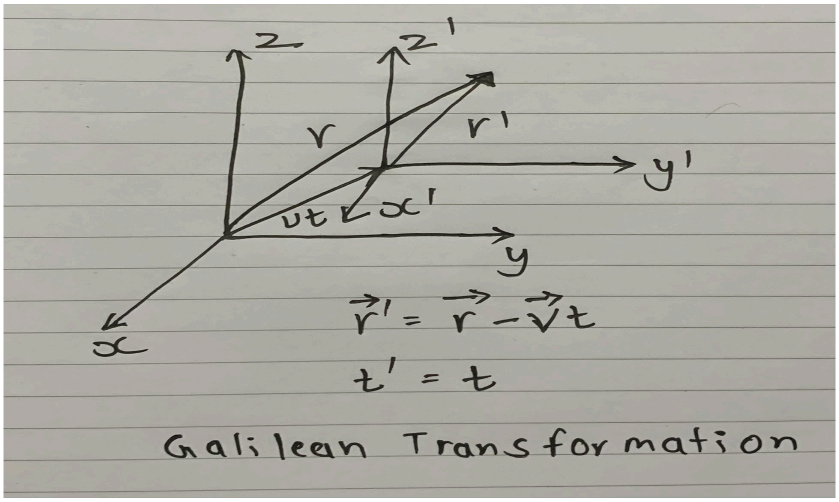


Spherical polar Coordinate System



Inertial Frames

The frames of reference which are stationary or moving with a constant velocity are called inertial frames of reference. These frames do not accelerate with respect to each other. There is no external force that is applied to the frame of reference. The typical example of inertial frames can be two passengers sitting on the bus, a spaceship that has stopped in deep space, and two people sitting on a chair. These frames are related to each other by Galilean Transformations. Since the two inertial frames can only have a uniform velocity with respect to each other, they follow Galilean Transformation. The Galilean Transformations for a single free particle in an inertial frame are given as t' = t - (1) and r' = r - $\bar{v}$t - (2). The transformation in terms of velocity can be given as $\bar{v}$' = $\bar{u}$ - $\bar{v}$ - (3), where $\bar{v}$' is the velocity of the particle in the primed frame, u is the velocity of the particle in the unprimed frame, and v is the relative velocity of the primed frame with respect to the unprimed frame. This covers the transformation of the inertial frames, now we move on to the non-inertial frames.



Non-inertial Frames

A non-inertial frame is a frame that has some acceleration or rotation with respect to an inertial frame of reference. With the introduction of non-inertial frames, there is also the introduction of a fictitious force or a pseudo force. These pseudo forces can be avoided and thus can be used to convert non-inertial frames into inertial frames. All non-inertial frames have a fictitious force associated with them. These forces are mainly present due to rotating reference frames. The forces arising from rotating reference frames are centrifugal force, Coriolis force, and Euler Force. Now we will derive all of these forces for non-inertial frames with respect to an inertial frame. First, we define an inertial frame with coordinates(x,y,z) and then we define a rotating reference frame with coordinates (x', y', z') with the same origin as that of the inertial frame. Le the rotation of the rotating frame is about the z-axis with a constant angular velocity ω. So we know that $d\theta/dt = \omega$ and z' = z, which can be inferred as $\theta(t) = \omega t + \theta_0$ for some value of θ_0 where θ(t) represents the angle between the x-y plane formed at time t by (x',y') and the x axis. Now for this case we assume that (x,y,z) = (x',y',z') at time t=0. Therefore we can take the value of $\theta_0 = 0$ or any multiple of 2π. Therefore the transformation of the rotating reference frame with respect to the inertial frame can be written as

x = x' cos(θ(t)) - y' sin(θ(t)) - (4)
y = x' cos(θ(t)) + y'sin(θ(t)) - (5)

And the reverse transformations can be written as

x' = x cos(-θ(t)) - y sin(-θ(t)) - (6)
y' = x cos(-θ(t) + y sin(-θ(t)) - (7)

The following are obtained by using a rotation matrix that rotates the frames by an angle of θ clockwise or anticlockwise depending on the frame we are rotating it about.

Now the relationship between velocities of the rotating reference frame with respect to the inertial frame can be written as

$\overline{v}$' = $\overline{v}$ + ωr - (8)

Where $\overline{v}$' is the velocity of the rotating reference frame, $\overline{v}$ is the velocity of the inertial frame such that $\omega = 0$, and ωr is just the angular velocity of the rotating reference frame.

Now we will derive the relationship between the acceleration between the rotating reference frame and the inertial frame

$d^2r/dt^2 = a + \omega \times (\omega \times r) + 2\omega \times v + d\omega/dt \times r$ - (9)

Where a is the actual acceleration of the rotating reference frame, $\omega \times (\omega \times r)$ is the centrifugal acceleration, $2\omega \times v$ is known as the coriolis acceleration, and $d\omega/dt \times r$ is called the Euler acceleration. Now that we have covered the transformation of the non-inertial frames with respect to inertial frames, we will now look at the forces which are produced due to the acceleration between the rotating reference frame and the inertial frame.

Forces on Non-Inertial Frames

There are four kinds of forces that act on non inertial frames. They are the inertial force, centrifugal force, Coriolis force, and Euler force. So the total force acting on the rotating reference frame with $\omega > 0$ is given as

$F_T = F_{Inertial} + F_{Centrifugal} + F_{Coriolis} + F_{Euler}$ - (10)

If $\omega = 0$ then only inertial force remains, which also is the condition of an inertial frame of reference as there is rotation present in the frame and thus Newton's First Law is verified. Now we will talk about each of these forces in detail.

Inertial Force

The force that is the physical property of a body is known as the inertial force of a body. It is a real force and it is always present in any given reference frame if there is an acceleration of the body. The inertial force is given by the equation

$$F = ma - (11)$$

Centrifugal Force

It is the first of the fictitious or pseudo forces that we will talk about for rotating reference frames. Centrifugal force occurs due to the centripetal acceleration in a rotating body which is towards the center. The centrifugal force also acts towards the center of the rotating reference frame with respect to the inertial frame. The equation for the centrifugal force can be given as

$$F_{Centrifugal} = m(\omega \times (\omega \times r)) - (12)$$

The example of a merry go round is the best for explaining the forces relating to the rotating reference frames. The simple magnitude of this force can be given by $m\omega^2 r$. Thus a person located on a merry go round will experience a centrifugal force irrespective of their location on the merry go round.

Coriolis Force

Coriolis Force is another fictitious or pseudo force which acts on a rotating reference frame with respect to an inertial frame. If the reference frame is rotating in a clockwise direction the coriolis force acts to the left of the motion of the object. And when the reference frame is rotating in an anticlockwise direction the coriolis force acts to the right to the motion of the object. The equation for coriolis force is given by the formula

$$F_{Coriolis} = m(2 \omega \times v) - (13)$$

Again as an example on the merry go round, imagine a person standing at the edge of a merry go round. The person is walking radially inward. The person is walking along the radial line at a speed v with respect to the merry go round. Here the ω vector points out of the plane. Then the coriolis force acts tangentially to the surface of the merry-go round.

Another example of the coriolis force is the Focault's Pendulum. A foucault pendulum shows that the Earth rotates. To understand the application of this pendulum we would need to consider a case where we can easily understand only the rotational motion of the earth and where the component of coriolis force is 0. For this we take the reference frame of the north pole. At the north pole any observer who is situated in space and is observing the plane of motion of the pendulum will experience that the Earth is rotating in a counter clockwise direction with respect to the plane of the pendulum. So to any observer who is situated on Earth the pendulum plane seems to rotate in a clockwise direction.

Euler Force

Euler Force can be best understood as a fictitious or pseudo tangential force. It only occurs when a rotating reference frame has a non-uniform rotation. It occurs due to the Euler acceleration as mentioned above. Euler Acceleration is also known as azimuthal acceleration or transverse acceleration. Again the best example of the Euler Force can be experienced on a merry go-round. The force that gives you a feeling that you are being dragged backward when the wheel starts to move and the feeling that you are being accelerated forward when the ride comes to a halt is the Euler Force. The formula for Euler Force can be written as

$F_{Euler} = d\omega/dt \times r$ - (14)

As it is clearly visible from the formula a person close to the outer perimeter of the merry go round will experience a greater force as compared to a person located at a location near the center of the merry go round.

Another example of the Euler Force can be understood with the help of an ice skater spinning on ice. The skater brings their arms close together which allows them to spin at a faster speed. Since the skater can be looked at from two frames the coriolis force is equal to the euler force as there is no net acceleration of the skater with respect to themselves. This also means that the skater is accelerating in a frame with respect to themselves with a tangential force which is also the Euler force for the skater.

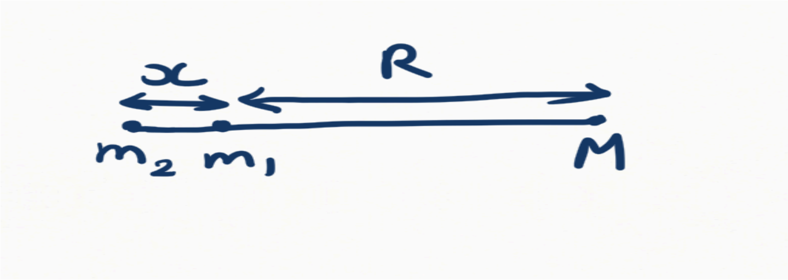
Understanding Practical Phenomenon using non-inertial frames

Tides

Tides are a natural phenomenon that occur on Earth due to the presence of the moon. It is due to the fact that gravitational force from the surface of a sphere like the Earth or the Moon is not uniform and thus variation in these forces along with the changing magnitude of these forces cause the oceans to bulge around the Earth creating the phenomenon of tides. To understand the General Tidal Force we first understand the Force through two special cases.

Longitudinal tidal force

To isolate the Tidal Effect we assume a system similar to the sun, moon and earth system, with two small masses m_1 and m_2 of the moon and earth respectively close to each other and a large mass M of the sun at a distance from these small masses. Here we further assume that the mass m_2 of the Earth is greater than that of m_1 which is the mass of the moon. We also assume that the distance between the earth and the moon is very small compared to the distance between earth and the sun and moon and the sun. We know that the masses m_1 and m_2 accelerate radially inwards towards M. We also assume that the masses m_1 and m_2 do not exert force on each other. Now we calculate the forces on m_1 with respect to the accelerating frame m_2. For that we assume a pseudo force that acts on mass m_1 along with the gravitational force which is applied on mass m_1. Therefore the total force on the mass m_1 can be calculated as



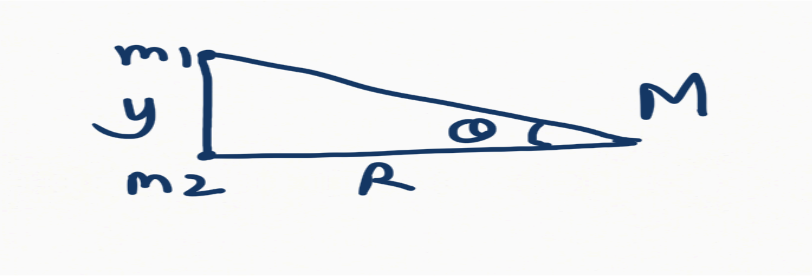
$$F_{Net} = F_{Gravitational} + F_{Pseudo}$$

By deriving the following equation where $F_{Gravitational} = m_1 GM/(R - x)^2 \hat{x}$ - (15)and $F_{Translational} = -m_1 GM/(R)^2 \hat{x}$ a - (16)

Now deriving the final equation for F_{Net} we have $F_{Net} = 2GMm_1x/R^3$ - (17)which is known as the tidal force.

Transverse Tidal Force

We assume the same criteria as above for the following arrangement of particles



Now we assume that the angle θ is very small and thus deriving the F_{Net} in the same way as the previous case we get $F_{net} = -Gm_1My\hat{y}/R^3$ - (18).

General Tidal Force

Now we can finally use both of these relations to write the General Tidal Force which can be written as $F_{Tidal} = GMm_1/R^3*(2x,-y)$ - (19) which is experienced on Earth due to the presence of the Moon.

Conclusion

Thus we can easily understand the effects of the forces acting on non-inertial frames and why it is necessary to understand the forces acting on these frames for practical phenomena.

Citations

Morin, D. (2019). *Introduction to classical mechanics: With problems and solutions*. Cambridge University Press.

Wikimedia Foundation. (2021, December 3). *Non-inertial reference frame*. Wikipedia. Retrieved May 25, 2022, from https://en.wikipedia.org/wiki/Non-inertial_reference_frame